Structure of the Vacuum in Nuclear Matter — A Nonperturbative Approach

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Abstract

We compute the vacuum polarisation correction to the binding energy of nuclear matter in the Walecka model using a nonperturbative approach. We first study such a contribution as arising from a ground state structure with baryon-antibaryon condensates. This yields the same results as obtained through the relativistic Hartree approximation of summing tadpole diagrams for the baryon propagator. Such a vacuum is then generalized to include quantum effects from meson fields through scalar-meson condensates. The method is applied to study properties of nuclear matter and leads to a softer equation of state giving a lower value of the incompressibility than would be reached without quantum effects. The density dependent effective sigma mass is also calculated including such vacuum polarisation effects.

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I. INTRODUCTION

Quantum Hadrodynamics (QHD) is a general framework for the nuclear many-body problem [1–3]. It is a renormalisable relativistic quantum field theory using hadronic degrees of freedom and has quite successfully described the properties of nuclear matter and finite nuclei. In the Walecka model (QHD-I) with nucleons interacting with scalar (σ) and vector (ω) mesons, it has been shown in the mean-field approximation that the saturation density and binding energy of nuclear matter may be fitted by adjusting the scalar and vector couplings [4]. This was first done by neglecting the Dirac sea and is called the no-sea approximation. In this approximation, several groups have investigated the effects of scalar self-interactions in nuclear matter [5] and finite nuclei [6] using a mean-field approach.

To include the sea effects, one does a self-consistent sum of tadpole diagrams for the baryon propagator [7]. This defines the relativistic Hartree approximation. There have also been calculations including corrections to the binding energy up to two-loops [8], which are seen to be rather large as compared to the one-loop results. However, it is seen that using phenomenological monopole form factors to account for the composite nature of the nucleons, such contribution is reduced substantially [9] so that it is smaller than the one-loop result. Recently, form factors have been introduced as a cure to the unphysical modes, the so called Landau poles [10], which one encounters while calculating the meson propagator as modified by the interacting baryon propagator of relativistic Hartree approximation. There have been also attempts to calculate the form factors by vertex corrections [11]. However, without inclusion of such form factors the mean-field theory is not stable against a perturbative loop expansion. This might be because the couplings involved here are too large (of order of 10) and the theory is not asymptotically free. Hence nonperturbative techniques need to be developed to consider nuclear many-body problems. The present work is a step in that direction including vacuum polarisation effects.

The approximation scheme here uses a squeezed coherent type of construction for the ground state [12,13] which amounts to an explicit vacuum realignment. The input here is equal-time quantum algebra for the field operators with a variational ansatz for the vacuum

structure and does not use any perturbative expansion or Feynman diagrams. We have earlier seen that this correctly yields the results of the Gross-Neveu model [14] as obtained by summing an infinite series of one-loop diagrams. We have also seen that it reproduces the gap equation in an effective QCD Hamiltonian [15] as obtained through the solution of the Schwinger-Dyson equations for the effective quark propagator. We here apply such a nonperturbative method to study the quantum vacuum in nuclear matter.

We organise the paper as follows. In section 2, we study the vacuum polarisation effects in nuclear matter as simulated through a vacuum realignment with baryon-antibaryon condensates. The condensate function is determined through a minimisation of the thermodynamic potential. The properties of nuclear matter as arising from such a vacuum are then studied and are seen to become identical to those obtained through the relativistic Hartree approximation. In section 3, we generalise such a vacuum state to include sigma condensates also, which are favoured with a quartic term in the sigma field in the Lagrangian. The quartic coupling is chosen to be positive which is necessary to consider vacuum polarisation effects from the sigma field. We also calculate the effective sigma mass arising through such quantum corrections as a function of density. The coupling here is chosen to give the value for the incompressibilty of nuclear matter in the correct range. Finally, in section 4, we summarise the results obtained through our nonperturbative approach and present an outlook.

II. VACUUM WITH BARYON AND ANTIBARYON CONDENSATES

We start with the Lagrangian density for the linear Walecka model given as

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - M - g_\sigma \sigma - g_\omega \gamma^\mu \omega_\mu)\psi + \frac{1}{2}\partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2}m_\sigma^2 \sigma^2 + \frac{1}{2}m_\omega^2 \omega^\mu \omega_\mu - \frac{1}{4}\omega^{\mu\nu}\omega_{\mu\nu}, \quad (1)$$

with

$$\omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu. \quad (2)$$

In the above, ψ, σ, and ω_μ are the fields for the nucleon, σ-, and ω-mesons with masses M, m_σ, and m_ω, respectively.

We use the mean-field approximation for the meson fields and retain the quantum nature of the fermion fields [14]. This amounts to taking meson fields as constant classical fields with translational invariance for nuclear matter. Thus we shall replace

$$g_\sigma \sigma \longrightarrow \langle g_\sigma \sigma \rangle \equiv g_\sigma \sigma_0 \tag{3a}$$

$$g_\omega \omega_\mu \longrightarrow \langle g_\omega \omega_\mu \rangle \equiv g_\omega \omega_\mu \delta^{\mu 0} = g_\omega \omega_0 \tag{3b}$$

where $\langle \cdots \rangle$ denotes the expectation value in nuclear matter and we have retained the zeroth component for the vector field to have nonzero expectation value.

The Hamiltonian density can then be written as

$$\mathcal{H} = \mathcal{H}_N + \mathcal{H}_\sigma + \mathcal{H}_\omega \tag{4}$$

with

$$\mathcal{H}_N = \psi^\dagger(-i\vec{\alpha} \cdot \vec{\bigtriangledown} + \beta M)\psi + g_\sigma \sigma \bar{\psi}\psi \tag{5a}$$

$$\mathcal{H}_\sigma = \frac{1}{2} m_\sigma^2 \sigma^2 \tag{5b}$$

$$\mathcal{H}_\omega = g_\omega \omega_0 \psi^\dagger \psi - \frac{1}{2} m_\omega^2 \omega_0^2 \tag{5c}$$

The equal-time quantization condition for the nucleons is given as

$$[\psi_\alpha(\vec{x},t), \psi_\beta^\dagger(\vec{y},t)]_+ = \delta_{\alpha\beta}\delta(\vec{x}-\vec{y}), \tag{6}$$

where α and β refer to the spin indices. We may now write down the field expansion for the nucleon field ψ at time $t = 0$ as given by [16]

$$\psi(\vec{x}) = \frac{1}{(2\pi)^{3/2}} \int \left[U_r(\vec{k}) c_{Ir}(\vec{k}) + V_s(-\vec{k}) \tilde{c}_{Is}(-\vec{k}) \right] e^{i\vec{k}\cdot\vec{x}} d\vec{k}, \tag{7}$$

with c_{Ir} and $\tilde{c}_{Is}$ as the baryon annihilation and antibaryon creation operators with spins r and s, respectively. In the above, U_r and V_s are given by

$$U_r(\vec{k}) = \begin{pmatrix} \cos\frac{\chi(\vec{k})}{2} \\ \vec{\sigma}\cdot\hat{k}\sin\frac{\chi(\vec{k})}{2} \end{pmatrix} u_{Ir}; \quad V_s(-\vec{k}) = \begin{pmatrix} -\vec{\sigma}\cdot\hat{k}\sin\frac{\chi(\vec{k})}{2} \\ \cos\frac{\chi(\vec{k})}{2} \end{pmatrix} v_{Is}, \tag{8}$$

For free massive fields $\cos\chi(\vec{k}) = M/\epsilon(\vec{k})$ and $\sin\chi(\vec{k}) = |\vec{k}|/\epsilon(\vec{k})$, with $\epsilon(\vec{k}) = \sqrt{\vec{k}^2 + M^2}$.

The above are consistent with the equal-time anticommutator algebra for the operators c and $\tilde{c}$ as given by

$$[c_{Ir}(\vec{k}), c_{Is}^\dagger(\vec{k}')]_+ = \delta_{rs}\delta(\vec{k}-\vec{k}') = [\tilde{c}_{Ir}(\vec{k}), \tilde{c}_{Is}^\dagger(\vec{k}')]_+. \tag{9}$$

The perturbative vacuum, say $\mid vac\rangle$, is defined through $c_{Ir}(\vec{k}) \mid vac\rangle = 0$ and $\tilde{c}_{Ir}^\dagger(\vec{k}) \mid vac\rangle = 0$.

To include the vacuum-polarisation effects, we shall now consider a trial state with baryon-antibaryon condensates. We thus explicitly take the ansatz for the above state as

$$\begin{aligned} |vac'\rangle &= \exp\Big[\int d\vec{k}\ f(\vec{k})\ c_{Ir}^\dagger(\vec{k})\ a_{rs}\tilde{c}_{Is}(-\vec{k}) - h.c.\Big]|vac\rangle \\ &\equiv U_F|vac\rangle, \end{aligned} \tag{10}$$

Here $a_{rs} = u_{Ir}^\dagger(\vec{\sigma}\cdot\hat{k})v_{Is}$ and $f(\vec{k})$ is a trial function associated with baryon-antibaryon condensates. We note that with the above transformation the operators corresponding to $\mid vac'\rangle$ are related to the operators corresponding to $|vac\rangle$ through the Bogoliubov transformation

$$\begin{pmatrix} d_I(\vec{k}) \\ \tilde{d}_I(-\vec{k}) \end{pmatrix} = \begin{pmatrix} \cos f(\vec{k}) & -\vec{\sigma}\cdot\hat{k}\sin f(\vec{k}) \\ \vec{\sigma}\cdot\hat{k}\sin f(\vec{k}) & \cos f(\vec{k}) \end{pmatrix} \begin{pmatrix} c_I(\vec{k}) \\ \tilde{c}_I(-\vec{k}) \end{pmatrix}, \tag{11}$$

for the nucleon.

We then use the method of thermofield dynamics [17] developed by Umezawa to construct the ground state for nuclear matter. We generalise the state with baryon-antibaryon condensates as given by (10) to finite temperature and density as [13]

$$|F(\beta)\rangle = U(\beta)|vac'\rangle \equiv U(\beta)U_F|vac\rangle. \tag{12}$$

The temperature-dependent unitary operator $U(\beta)$ is given as [17]

$$U(\beta) = \exp\left(B^\dagger(\beta) - B(\beta)\right), \tag{13}$$

with

$$B^\dagger(\beta) = \int d\vec{k} \, \Big[\theta_-(\vec{k},\beta) \; d^\dagger_{Ir}(\vec{k}) \; \underline{d}^\dagger_{Ir}(-\vec{k}) + \theta_+(\vec{k},\beta) \; \tilde{d}_{Ir}(\vec{k}) \; \underline{\tilde{d}}_{Ir}(-\vec{k}) \Big]. \tag{14}$$

The underlined operators are the operators corresponding to the doubling of the Hilbert space that arises in thermofield dynamics method. We shall determine the condensate function $f(\vec{k})$, and the functions $\theta_-(\vec{k},\beta)$ and $\theta_+(\vec{k},\beta)$ of the thermal vacuum through minimisation of the thermodynamic potential. To evaluate the expectation value of the energy density with respect to the thermal vacuum, we shall use the following formula.

$$\langle \psi^\dagger_\gamma(\vec{x}) \psi_\delta(\vec{y}) \rangle_\beta = \frac{1}{(2\pi)^3} \int \Big(\Lambda_-(\vec{k},\beta) \Big)_{\delta\gamma} e^{-i\vec{k}.(\vec{x}-\vec{y})} d\vec{k}, \tag{15}$$

where

$$\begin{aligned} \Lambda_-(\vec{k},\beta) = \frac{1}{2}\Big[&(\cos^2\theta_+ + \sin^2\theta_-) - \Big(\gamma^0 \cos(\chi(\vec{k}) - 2f(\vec{k})) \\ &+ \vec{\alpha}\cdot\hat{k} \;\; \sin(\chi(\vec{k}) - 2f(\vec{k})) \Big) (\cos^2\theta_+ - \sin^2\theta_-) \Big]. \end{aligned} \tag{16}$$

We now proceed to calculate the energy density,

$$\epsilon \equiv \langle \mathcal{H} \rangle_\beta = \epsilon_N + \epsilon_\sigma + \epsilon_\omega \tag{17}$$

with

$$\epsilon_N = -\frac{\gamma}{(2\pi)^3} \int d\vec{k} \Big[\epsilon(\vec{k}) \cos 2f(\vec{k}) - \frac{g_\sigma \sigma_0}{\epsilon(k)} \Big(M \cos 2f(\vec{k}) + |\vec{k}| \sin 2f(\vec{k}) \Big) \Big] (\cos^2\theta_+ - \sin^2\theta_-) \tag{18a}$$

$$\epsilon_\sigma = \frac{1}{2} m_\sigma^2 \sigma_0^2, \tag{18b}$$

and

$$\epsilon_\omega = g_\omega \omega_0 \; \gamma (2\pi)^{-3} \int d\vec{k} (\cos^2\theta_+ + \sin^2\theta_-) - \frac{1}{2} m_\omega^2 \omega_0^2. \tag{18c}$$

The thermodynamic potential is then given as

$$\Omega = \epsilon - \frac{1}{\beta} \mathcal{S} - \mu \rho_B, \tag{19}$$

with the entropy density

$$\mathcal{S} = -\gamma(2\pi)^{-3}\int d\vec{k}\Big[\sin^2\theta_- \ln(\sin^2\theta_-) + \cos^2\theta_- \ln(\cos^2\theta_-)$$
$$+ \sin^2\theta_+ \ln(\sin^2\theta_+) + \cos^2\theta_+ \ln(\cos^2\theta_+)\Big] + \mathcal{S}_\sigma + \mathcal{S}_\omega \tag{20}$$

and the baryon density

$$\rho_B = \gamma(2\pi)^{-3}\int d\vec{k}(\cos^2\theta_+ + \sin^2\theta_-). \tag{21}$$

In the above, γ is the spin isospin degeneracy factor and is equal to 4 for nuclear matter. Further, $\mathcal{S}_\sigma$ and $\mathcal{S}_\omega$ are the contributions to the entropy density from σ- and ω-mesons, respectively. It may be noted here that these are independent of the functions $f(\vec{k})$, $\theta_\pm(\vec{k},\beta)$ associated with the nucleons and hence are not relevant for the nuclear matter properties at zero temperature. Extremising the thermodynamic potential Ω with respect to the condensate function $f(\vec{k})$ and the functions $\theta_\mp$ yields

$$\tan 2f(\vec{k}) = \frac{g_\sigma\sigma_0|\vec{k}|}{\epsilon(k)^2 + Mg_\sigma\sigma_0} \tag{22}$$

and

$$\sin^2\theta_\mp = \frac{1}{\exp(\beta(\epsilon^*(k)\mp\mu^*))+1} \tag{23}$$

with $\epsilon^*(k) = (k^2 + M^{*2})^{1/2}$ and $\mu^* = \mu - g_\omega\omega_0$ as the effective energy density and effective chemical potential, where the effective nucleon mass is $M^* = M + g_\sigma\sigma_0$.

Then the expression for the energy density becomes

$$\epsilon = \epsilon_N + \epsilon_\sigma + \epsilon_\omega, \tag{24}$$

with

$$\epsilon_N = \gamma(2\pi)^{-3}\int d\vec{k}(k^2 + M^{*2})^{1/2}(\sin^2\theta_- - \cos^2\theta_+), \tag{25a}$$

$$\epsilon_\sigma = \frac{1}{2}m_\sigma^2\sigma_0^2, \tag{25b}$$

and

$$\epsilon_\omega = g_\omega \omega_0 \; \gamma (2\pi)^{-3} \int d\vec{k} (\sin^2\theta_- + \cos^2\theta_+) - \frac{1}{2} m_\omega^2 \omega_0^2. \tag{25c}$$

We now proceed to study the properties of nuclear matter at zero temperature. In that limit the distribution functions for the baryons and antibaryons are given as

$$\sin^2\theta_- = \Theta(\mu^* - \epsilon^*(\vec{k})); \qquad\qquad \sin^2\theta_+ = 0. \tag{26}$$

The energy density after subtracting out the pure vacuum contribution then becomes

$$\begin{aligned} \epsilon_0 &\equiv \epsilon(\theta_-, f) - \epsilon(\theta_- = 0, f = 0) \\ &= \epsilon_{MFT} + \Delta\epsilon \end{aligned} \tag{27}$$

with

$$\epsilon_{MFT} = \gamma (2\pi)^{-3} \int\limits_{|\vec{k}| < k_F} d\vec{k} (k^2 + M^{*2})^{1/2} + \frac{1}{2} m_\sigma^2 \sigma_0^2 + g_\omega \omega_0 \rho_B - \frac{1}{2} m_\omega^2 \omega_0^2 \tag{28}$$

and

$$\Delta\epsilon = -\gamma (2\pi)^{-3} \int d\vec{k} \Big[(k^2 + M^{*2})^{1/2} - (k^2 + M^2)^{1/2} - \frac{g_\sigma \sigma_0 M}{(k^2 + M^2)^{1/2}} \Big]. \tag{29}$$

The above expression for the energy density is divergent. It is renormalised [7] by adding the counter terms

$$\epsilon_{ct} = \sum_{n=1}^{4} C_n \sigma_0^n. \tag{30}$$

The addition of the counter term linear in σ_0 amounts to normal ordering of the scalar density in the perturbative vacuum and cancels exactly with the last term in equation (29) [7]. The first two terms of the same equation corresponds to the shift in the Dirac sea arising from the change in the nucleon mass at finite density when σ acquires a vacuum expectation value, and consequent divergences cancel with the counter terms of (30) with higher powers in σ_0 [7]. Then we have the expression for the finite renormalised energy density

$$\epsilon_{ren} = \epsilon_{MFT} + \Delta\epsilon_{ren}, \tag{31}$$

where

$$\Delta\epsilon_{ren} = -\frac{\gamma}{16\pi^2}\Big[M^{*4}\ln(\frac{M^*}{M}) + M^3(M-M^*) - \frac{7}{2}M^2(M-M^*)^2 + \frac{13}{3}M(M-M^*)^3 - \frac{25}{12}M(M-M^*)^4\Big]. \quad (32)$$

For a given baryon density as given by

$$\rho_B = \gamma(2\pi)^{-3}\int d\vec{k}\Theta(k_F - k), \quad (33)$$

the thermodynamic potential given by equation (19) is now finite and is a function of σ_0 and ω_0. This when minimised with respect to σ_0 gives the self-consistency condition for the effective nucleon mass,

$$M^* = M - \frac{g_\sigma^2}{m_\sigma^2}\frac{\gamma}{(2\pi)^3}\int d\vec{k}\frac{M^*}{\epsilon(k)^*}\Theta(k_F - k) + \Delta M^* \quad (34)$$

where

$$\Delta M^* = \frac{g_\sigma^2}{m_\sigma^2}\frac{\gamma}{(2\pi)^3}\Big[M^{*3}\ln(\frac{M^*}{M}) + M^2(M-M^*) - \frac{5}{2}M^2(M-M^*)^2 + \frac{11}{6}M(M-M^*)^3\Big] \quad (35)$$

We note that the self-consistency condition for the effective nucleon mass as well as the energy density as obtained here through an explicit construct of a state with baryon-antibaryon condensates are identical to those obtained through summing tadpole diagrams for the baryon propagator in the relativistic Hartree approximation [7].

III. ANSATZ STATE WITH BARYON ANTIBARYON AND SIGMA CONDENSATES

We next consider the quantum corrections due to the scalar mesons as arising from a vacuum realignment with sigma condensates. This means that the σ-field is no longer classical, but is now treated as a quantum field. As will be seen later, a quartic term in the sigma field would favour such condensates. Self-interactions of scalar fields with cubic and quartic terms have been considered earlier [18] in the no-sea approximation [6] as well as including the quantum corrections arising from the sigma fields [1,19,20]. They may be regarded as mediating three- and four-body interactions between the nucleons. The best

fits to incompressibility in nuclear matter, single-particle spectra and properties of deformed nuclei are achieved with a negative value for the quartic coupling in the sigma field. However, with such a negative coupling the energy spectrum of the theory becomes unbounded from below [21] for large σ and hence it is impossible to study excited spectra or to include vacuum polarisation effects.

Including a quartic scalar self-interaction, eq. (5b) is modified to

$$\mathcal{H}_\sigma = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}m_\sigma^2\sigma^2 + \lambda\sigma^4, \tag{36}$$

with m_σ and λ being the bare mass and coupling constant respectively. The σ field satisfies the quantum algebra

$$[\sigma(\vec{x}), \dot{\sigma}(\vec{y})] = i\delta(\vec{x} - \vec{y}). \tag{37}$$

We may expand the field operators in terms of creation and annihilation operators at time $t = 0$ as

$$\sigma(\vec{x}, 0) = \frac{1}{(2\pi)^{3/2}} \int \frac{d\vec{k}}{\sqrt{2\omega(\vec{k})}} \left(a(\vec{k}) + a^\dagger(-\vec{k}) \right) e^{i\vec{k}\cdot\vec{x}}, \tag{38a}$$

$$\dot{\sigma}(\vec{x}, 0) = \frac{i}{(2\pi)^{3/2}} \int d\vec{k} \sqrt{\frac{\omega(\vec{k})}{2}} \left(-a(\vec{k}) + a^\dagger(-\vec{k}) \right) e^{i\vec{k}\cdot\vec{x}}. \tag{38b}$$

In the above, $\omega(\vec{k})$ is an arbitrary function which for free fields is given by $\omega(\vec{k}) = \sqrt{\vec{k}^2 + m_\sigma^2}$ and the perturbative vacuum is defined corresponding to this basis through $a \mid vac\rangle = 0$. The expansions (38) and the quantum algebra (37) yield the commutation relation for the operators a as

$$[a(\vec{k}), a^\dagger(\vec{k}')] \; = \; \delta(\vec{k} - \vec{k}'). \tag{39}$$

As seen in the previous section a realignment of the ground state from $\mid vac\rangle$ to $\mid vac'\rangle$ with nucleon condensates amounts to including quantum effects. We shall adopt a similar procedure now to calculate the quantum corrections arising from the σ-field. We thus modify the ansatz for the trial ground state as given by (10) to include σ condensates as [13]

$$|\Omega\rangle = U_\sigma U_F |vac\rangle, \tag{40}$$

with

$$U_\sigma = U_{II} U_I \tag{41}$$

where $U_i = \exp(B_i^\dagger \; - \; B_i)$, $(i = I, II)$. Explicitly the B_i are given as

$$B_I^\dagger = \int d\vec{k} \sqrt{\frac{\omega(\vec{k})}{2}} f_\sigma(\vec{k}) a^\dagger(\vec{k}), \tag{42a}$$

and

$$B_{II}{}^\dagger = \frac{1}{2} \int d\vec{k} g(\vec{k}) a'^\dagger(\vec{k}) a'^\dagger(-\vec{k}). \tag{42b}$$

In the above, $a'(\vec{k}) = U_I a(\vec{k}) U_I^{-1} = a(\vec{k}) - \sqrt{\frac{\omega(\vec{k})}{2}} f_\sigma(\vec{k})$ corresponds to a shifted field operator associated with the coherent state [13] and satisfies the same quantum algebra as given in eq. (39). Thus in this construct for the ground state we have two functions $f_\sigma(\vec{k})$ and $g(\vec{k})$ which will be determined through minimisation of energy density. Further, since $\mid \Omega\rangle$ contains an arbitrary number of $a'^\dagger$ quanta, $a' \mid \Omega\rangle \neq 0$. However, we can define the basis $b(\vec{k})$, $b^\dagger(\vec{k})$ corresponding to $\mid \Omega\rangle$ through the Bogoliubov transformation as

$$\begin{pmatrix} b(\vec{k}) \\ b^\dagger(-\vec{k}) \end{pmatrix} = U_{II} \begin{pmatrix} a'(\vec{k}) \\ a'^\dagger(-\vec{k}) \end{pmatrix} U_{II}^{-1} = \begin{pmatrix} \cosh g & -\sinh g \\ -\sinh g & \cosh g \end{pmatrix} \begin{pmatrix} a'(\vec{k}) \\ a'^\dagger(-\vec{k}) \end{pmatrix}. \tag{43}$$

It is easy to check that $b(\vec{k}) \mid \Omega\rangle = 0$. Further, to preserve translational invariance $f_\sigma(\vec{k})$ has to be proportional to $\delta(\vec{k})$ and we take $f_\sigma(\vec{k}) = \sigma_0 (2\pi)^{3/2} \delta(\vec{k})$. σ_0 will correspond to a classical field of the conventional approach [13]. We next calculate the expectation value of the Hamiltonian density for the σ-meson given by equation(36). Using the transformations (43) it is easy to evaluate that

$$\langle \Omega \mid \sigma \mid \Omega \rangle = \sigma_0, \tag{44a}$$

but,

$$\langle \Omega \mid \sigma^2 \mid \Omega \rangle = {\sigma_0}^2 + I, \tag{44b}$$

where

$$I = \frac{1}{(2\pi)^3} \int \frac{d\vec{k}}{2\,\omega(k)} (\cosh 2g + \sinh 2g). \tag{44c}$$

Using equations (36) and (44) the energy density of $\mathcal{H}_\sigma$ with respect to the trial state becomes [13]

$$\begin{aligned} \epsilon_\sigma \equiv \langle \Omega \mid \mathcal{H}_\sigma \mid \Omega \rangle = & \frac{1}{2} \frac{1}{(2\pi)^3} \int \frac{d\vec{k}}{2\omega(k)} \Big[k^2 (\sinh 2g + \cosh 2g) + \omega^2(k)(\cosh 2g - \sinh 2g) \Big] \\ & + \frac{1}{2} m_\sigma^2 I + 6\lambda\sigma_0^2 I + 3\lambda I^2 + \frac{1}{2} m_\sigma^2 \sigma_0^2 + \lambda \sigma_0^4. \end{aligned} \tag{45}$$

Extremising the above energy density with respect to the function $g(k)$ yields

$$\tanh 2g(k) = -\frac{6\lambda I + 6\lambda {\sigma_0}^2}{\omega(k)^2 + 6\lambda I + 6\lambda {\sigma_0}^2}. \tag{46}$$

It is clear from the above equation that in the absence of a quartic coupling no such condensates are favoured since the condensate function vanishes for $\lambda = 0$. Now substituting this value of $g(k)$ in the expression for the σ-meson energy density yields

$$\epsilon_\sigma = \frac{1}{2} m_\sigma^2 {\sigma_0}^2 + \lambda {\sigma_0}^4 + \frac{1}{2} \frac{1}{(2\pi)^3} \int d\vec{k} (k^2 + M_\sigma^2)^{1/2} - 3\lambda I^2 \tag{47}$$

where

$$M_\sigma^2 = m_\sigma^2 + 12\lambda I + 12\lambda \sigma_0^2 \tag{48}$$

with

$$I = \frac{1}{(2\pi)^3} \int \frac{d\vec{k}}{2} \frac{1}{(\vec{k}^2 + M_\sigma^2)^{1/2}} \tag{49}$$

obtained from equation (44c) after substituting for the condensate function $g(k)$ as in equation (46). The expression for the "effective potential" ϵ_σ contains divergent integrals. Since our approximation is nonperturbatively self-consistent, the field-dependent effective mass M_σ is also not well defined because of the infinities in the integral I given by equation (49). Therefore we first obtain a well-defined finite expression for M_σ by renormalisation. We use the renormalisation prescription of ref. [22] and thus obtain the renormalised mass m_R and coupling λ_R through

$$\frac{m_R^2}{\lambda_R} = \frac{m^2}{\lambda} + 12 I_1(\Lambda), \tag{50a}$$

$$\frac{1}{\lambda_R} = \frac{1}{\lambda} + 12 I_2(\Lambda, \mu), \tag{50b}$$

where I_1 and I_2 are the integrals

$$I_1(\Lambda) = \frac{1}{(2\pi)^3} \int\limits_{|\vec{k}|<\Lambda} \frac{d\vec{k}}{2k}, \tag{51a}$$

$$I_2(\Lambda, \mu) = \frac{1}{\mu^2} \int\limits_{|\vec{k}|<\Lambda} \frac{d\vec{k}}{(2\pi)^3} \Big(\frac{1}{2k} - \frac{1}{2\sqrt{k^2+\mu^2}} \Big), \tag{51b}$$

with μ as the renormalisation scale and Λ as an ultraviolet momentum cut-off. It may be noted here that with the use of the above renormalisation prescription the effective sigma mass M_σ and the energy density ultimately become independent of Λ and stay finite in the limit $\Lambda \to \infty$. Using equations (50a) and (50b) in equation (48), we have the gap equation for M_σ^2 in terms of the renormalised parameters as

$$M_\sigma^2 = m_R^2 + 12\lambda_R \sigma_0^2 + 12\lambda_R I_f(M_\sigma), \tag{52}$$

where

$$I_f(M_\sigma) = \frac{M_\sigma^2}{16\pi^2} \ln\Big(\frac{M_\sigma^2}{\mu^2}\Big). \tag{53}$$

Then using the above equations we simplify equation (47) to obtain the energy density for the σ in terms of σ_0 as

$$\epsilon_\sigma = 3\lambda_R \Big(\sigma_0^2 + \frac{m_R^2}{12\lambda_R}\Big)^2 + \frac{M_\sigma^4}{64\pi^2}\Bigg(\ln\Big(\frac{M_\sigma^2}{\mu^2}\Big) - \frac{1}{2}\Bigg) - 3\lambda_R I_f^2 - 2\lambda\sigma_0^4. \tag{54}$$

The above expression is given in terms of the renormalized σ mass m_R and the renormalized coupling λ_R except for the last term which is still in terms of the bare coupling constant λ and did not get renormalised because of the structure of the gap equation [23]. However, from the renormalisation condition (50b) it is easy to see that when λ_R is kept fixed, as the ultraviolet cut-off Λ in eq. (51b) goes to infinity, the bare coupling $\lambda \to 0_-$. Therefore the last term in eq. (54) will be neglected in the numerical calculations.

After subtracting the vacuum contribution, we get

$$\begin{aligned}\Delta\epsilon_\sigma &= \epsilon_\sigma - \epsilon_\sigma(\sigma_0=0)\\ &= \frac{1}{2}m_R^2\sigma_0^2 + 3\lambda_R\sigma_0^4 + \frac{M_\sigma^4}{64\pi^2}\Bigg(\ln\Big(\frac{M_\sigma^2}{\mu^2}\Big) - \frac{1}{2}\Bigg) - 3\lambda_R I_f^2\\ &- \frac{M_{\sigma,0}^4}{64\pi^2}\Bigg(\ln\Big(\frac{M_{\sigma,0}^2}{\mu^2}\Big) - \frac{1}{2}\Bigg) + 3\lambda_R I_{f0}^2,\end{aligned} \tag{55}$$

where $M_{\sigma,0}$ and I_{f0} are the expressions as given by eqs. (52) and (53) with $\sigma_0 = 0$.

In the limit of the coupling, $\lambda_R = 0$, one can see that eq. (55) reduces to eq. (25b) as it should. Also, we note that the sign of λ_R must be chosen to be positive, because otherwise the energy density would become unbounded from below with vacuum fluctuations [3,20,21].

The expectation value for the energy density after subtracting out the vacuum contribution as given by eq. (27), now with sigma condensates is modified to

$$\epsilon_0 = \epsilon_0^{finite} + \Delta\epsilon, \tag{56}$$

where

$$\epsilon_0^{finite} = \gamma(2\pi)^{-3}\int_{|\vec{k}|<k_F} d\vec{k}(k^2+M^{*2})^{1/2} + g_\omega\omega_0\rho_B - \frac{1}{2}m_\omega^2\omega_0^2 + \Delta\epsilon_\sigma, \tag{57}$$

with $\Delta\epsilon_\sigma$ given through eq. (55) and $\Delta\epsilon$ is the divergent part of the energy density given by equation (29). We renormalise by adding the same counter terms as given by (30) so that as earlier the renormalised mass and the renormalised quartic coupling remain unchanged [1]. This yields the expression for the energy density

$$\epsilon_{ren} = \epsilon_0^{finite} + \Delta\epsilon_{ren}, \tag{58}$$

with $\Delta\epsilon_{ren}$ given by eq. (32). As earlier the energy density is to be minimised with respect to σ_0 to obtain the optimised value for σ_0, thus determining the effective mass M^* in a self-consistent manner.

The energy density from the σ field as given by eq. (55) is still in terms of the renormalisation scale μ which is arbitrary. We choose this to be equal to the renormalised sigma mass m_R in doing the numerical calculations. This is because changing μ would mean changing the quartic coupling λ_R, and λ_R here enters as a parameter to be chosen to give the incompressibility in the correct range.

For a given baryon density, ρ_B, the binding energy for nuclear matter is

$$E_B = \epsilon_{ren}/\rho_B - M, \tag{59}$$

The parameters g_σ, g_ω and λ_R are fitted so as to describe the ground-state properties of nuclear matter correctly. We discuss the results in the next section.

IV. RESULTS AND DISCUSSIONS

We now proceed with the numerical calculations to study the nuclear matter properties at zero temperature. We take the nucleon and ω-meson masses to be their experimental values as 939 MeV and 783 MeV. We first calculate the binding energy per nucleon as given in equation (59) and fit the scalar and vector couplings g_σ and g_ω to get the correct saturation properties of nuclear matter. This involves first minimising the energy density in eq. (58) with respect to σ_0 to get the optimised scalar field ground state expectation value σ_{min}. This procedure also naturally includes obtaining the in-medium σ-meson mass M_σ through solving the gap equation (52) in a self-consistent manner. Obtaining the optimized σ_{min} amounts to getting the effective nucleon mass $M^* = M + g_\sigma \sigma_{min}$. We fix the meson couplings from the saturation properties of the nuclear matter for given renormalised sigma mass and coupling m_R and λ_R. Taking $m_R = 520$ MeV, the values of g_σ and g_ω are 7.34 and 8.21 for $\lambda_R = 1.8$, and are 6.67 and 7.08 for $\lambda_R = 5$ respectively. Using these values, we calculate the binding energy for nuclear matter as a function of the Fermi momentum and plot it in fig. 1. In the same figure we also plot the results for the relativistic Hartree and for the no-sea approximation. Clearly, including baryon and σ-meson quantum corrections leads to a softer equation of state and the softening increases for higher values of λ_R. The incompressibility of the nuclear matter is given as [24]

$$K = k_F^2 \frac{\partial^2 \epsilon}{\partial k_F^2} \tag{60}$$

evaluated at the saturation Fermi momentum. The value of K is found to be 401 MeV for $\lambda_R = 1.8$ and 329 MeV for $\lambda_R = 5$. These are smaller than the mean-field result of 545 MeV

[4], as well as that of relativistic Hartree of 450 MeV [7] and are similar to those obtained in ref. [20] containing cubic and quartic self-interaction of the σ-meson.

In fig. 2, we plot the effective nucleon mass $M^* = M + g_\sigma \sigma_{min}$ as a function of Fermi momentum with σ_{min} obtained from the minimisation of the energy density in a self-consistent manner. At the saturation density of $k_F = 1.42$ fm^{-1}, we get $M^* = 0.752M$ and $0.815M$ for $\lambda_R = 1.8$ and $\lambda_R = 5$, respectively. These values may be compared with the results of $M^* = 0.56M$ in the no-sea approximation and of $0.72M$ in the relativistic Hartree.

In fig. 3, we plot the vector and the scalar potentials as functions of k_F for sigma self coupling $\lambda_R = 1.8$ and 5. At saturation density the scalar and vector contributions are $U_S \equiv g_\sigma \sigma_{min} = -232.7$ MeV and $U_V \equiv g_\omega \omega_0 = 163.4$ MeV for $\lambda_R = 1.8$ and are -173.14 MeV and 107.74 MeV for $\lambda_R = 5$ respectively. These give rise to the nucleon potential $(U_S + U_V)$ of -69.3 MeV and -65.4 MeV and an antinucleon potential $(U_S - U_V)$ of -396.1 MeV and -280.9 MeV for $\lambda_R = 1.8$ and 5. Clearly the inclusion of the quantum corrections reduces the antinucleon potential as compared to both the relativistic Hartree (-450 MeV) [7] and the no-sea results (-746 MeV) [4].

In fig. 4, we plot the in-medium σ-meson mass M_σ of eq. (52) as a function of baryon density for $\lambda_R = 1.8$ and 5. M_σ increases with density as λ_R is positive and the magnitude of σ_{min} increases with density too. However, the change in M_σ is rather small.

In fig. 5, we plot the incompressibility K as a function of the quartic coupling λ_R for different values of m_R, the renormalised sigma mass in vacuum. The value of K decreases with increase in λ_R similar to the results obtained in ref. [20]. In fig. 6, we plot the effective nucleon mass versus the sigma self coupling for various values of m_R. The value of M^* increases with λ_R, which is a reflection of the diminishing nucleon-sigma coupling strength for larger values of the quartic self-interaction.

To summarise we have used a nonperturbative approach to include quantum effects in nuclear matter using the framework of QHD. Instead of going through a loop expansion and summing over an infinite series of Feynman diagrams we have included the quantum corrections through a realignment of the ground state with baryon as well as meson condensates. It is interesting to note that inclusion of baryon-antibaryon condensates with the particular

ansatz determined through minimisation of the thermodynamic potential yields the same results as obtained in the relativistic Hartree approximation. This results in a softer equation of state as compared to the no-sea approximation. Inclusion of scalar meson quantum corrections in a self-consistent manner leads to a further softening of the equation of state. The value for the incompressibility of nuclear matter is within the range of 200–400 MeV [25]. It is known that most of the parameter sets which explain the ground state properties of nuclear matter and finite nuclei quite well are with a negative quartic coupling. But the energy spectrum in such a case is unbounded from below [21] for large σ thus making it impossible to include vacuum polarisation effects. We have included the quantum effects with a quartic self interaction through sigma condensates taking the coupling to be positive. We have also calculated the effective mass of the sigma field as modified by the quantum corrections from baryon and sigma fields. The effective sigma mass is seen to increase with density.

We have also looked at the behaviour of the incompressibility as a function of the coupling λ_R for various values of sigma mass, which is seen to decrease with the coupling. Finally, we have looked at the effect of the sigma quartic coupling on the effective nucleon mass which grows with the coupling. Generally, higher values of the quartic term in the potential of the σ-meson tend to reduce the large meson fields and thus the strong relativistic effects in the nucleon sector. Clearly, the approximation here lies in the specific ansatz for the ground-state structure. However, a systematic inclusion of more general condensates than the pairing one as used here might be an improvement over the present one. The method can also be generalised to finite temperature as well as to finite nuclei, e.g., using the local density approximation. Work in this direction is in progress.

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Figures Captions

Fig. 1. The binding energy for nuclear matter as a function of Fermi momentum k_F corresponding to the no-sea and the relativistic Hartree approximations and the approach including quantum corrections from baryon and σ-meson given by eq. (59). It is seen that the equation of state is softer with such quantum corrections.

Fig. 2. The effective nucleon mass for nuclear matter as a function of the Fermi momentum, k_F.

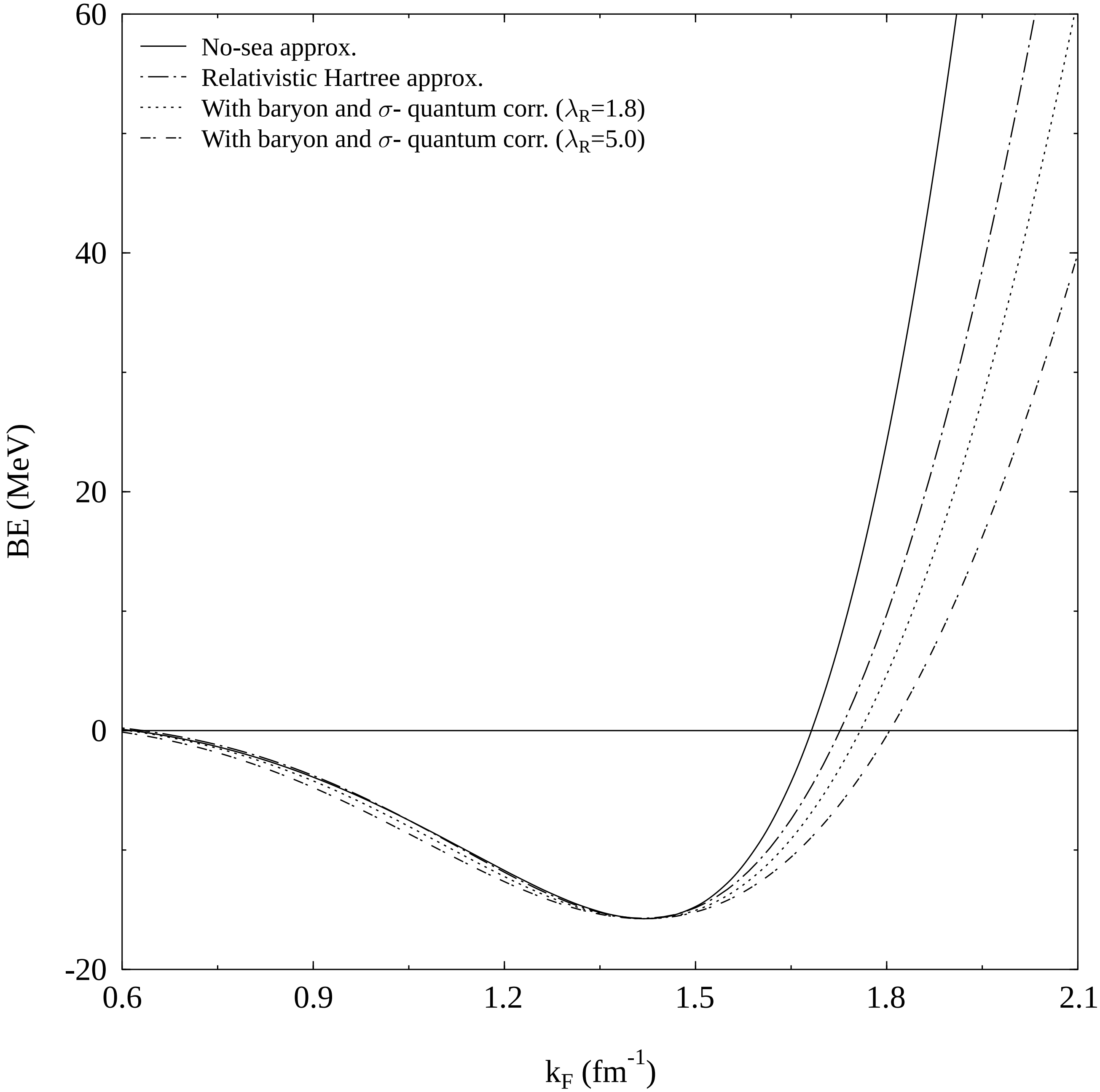
Fig. 3. The scalar potential U_S (negative values) and vector potential U_V (positive values) for nuclear matter as functions of Fermi momentum k_F.

Fig. 4. The in-medium σ-meson mass M_σ of eq. (52) as a function of density, which is seen to increase with density. However, the change is seen to be rather small.

Fig. 5. The incompressibility K versus the quartic coupling λ_R for various values of m_R, which is seen to decrease with λ_R. The values of K are higher for larger values of the sigma mass.

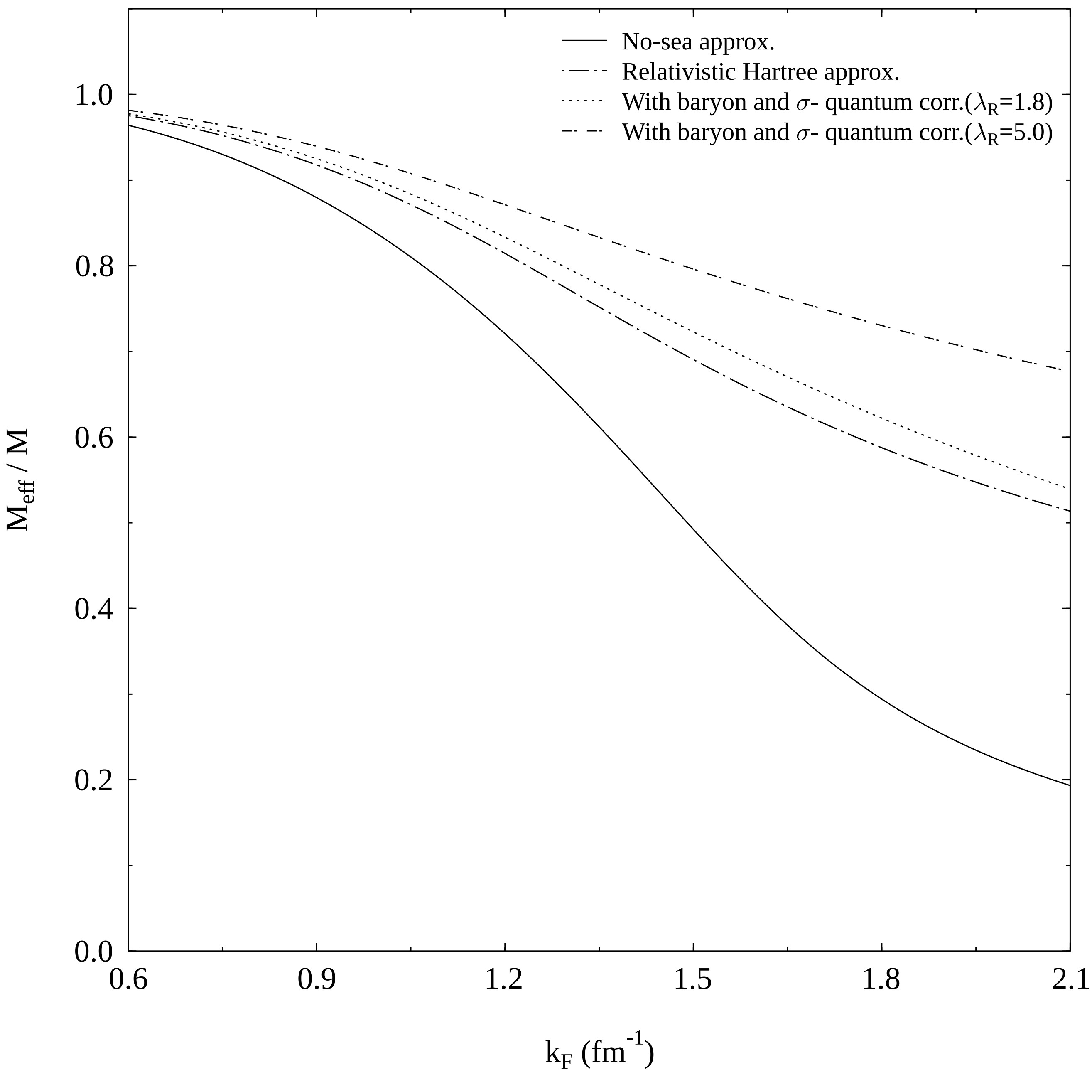
Fig. 6. The effective nucleon mass as a function of λ_R for different values of m_R. It is seen to decrease with increase in the coupling.

Figure 1



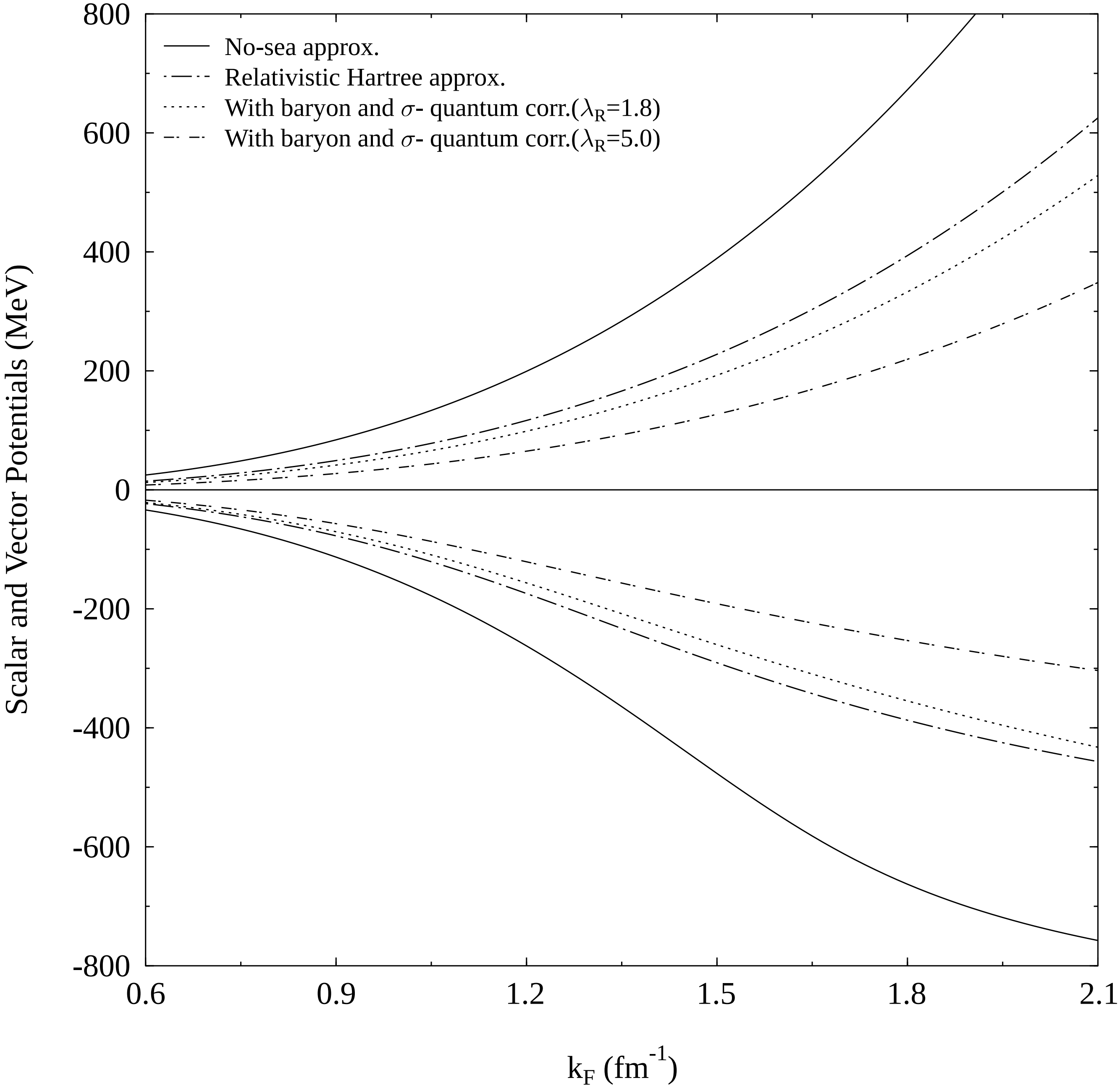
60
40
20
0
-20
0.6
0.9
1.2
1.5
1.8
2.1
BE (MeV)
k_F (fm^{-1})
No-sea approx.
Relativistic Hartree approx.
With baryon and σ- quantum corr. (λ_R=1.8)
With baryon and σ- quantum corr. (λ_R=5.0)

Figure 2



No-sea approx.
Relativistic Hartree approx.
With baryon and σ- quantum corr.(λ_R=1.8)
With baryon and σ- quantum corr.(λ_R=5.0)
1.0
0.8
0.6
0.4
0.2
0.0
0.6
0.9
1.2
1.5
1.8
2.1
M_{eff} / M
k_F (fm^{-1})

Figure 3



No-sea approx.
Relativistic Hartree approx.
With baryon and σ- quantum corr.(λR=1.8)
With baryon and σ- quantum corr.(λR=5.0)
Scalar and Vector Potentials (MeV)
800
600
400
200
0
-200
-400
-600
-800
0.6
0.9
1.2
1.5
1.8
2.1
kF (fm-1)

Figure 4

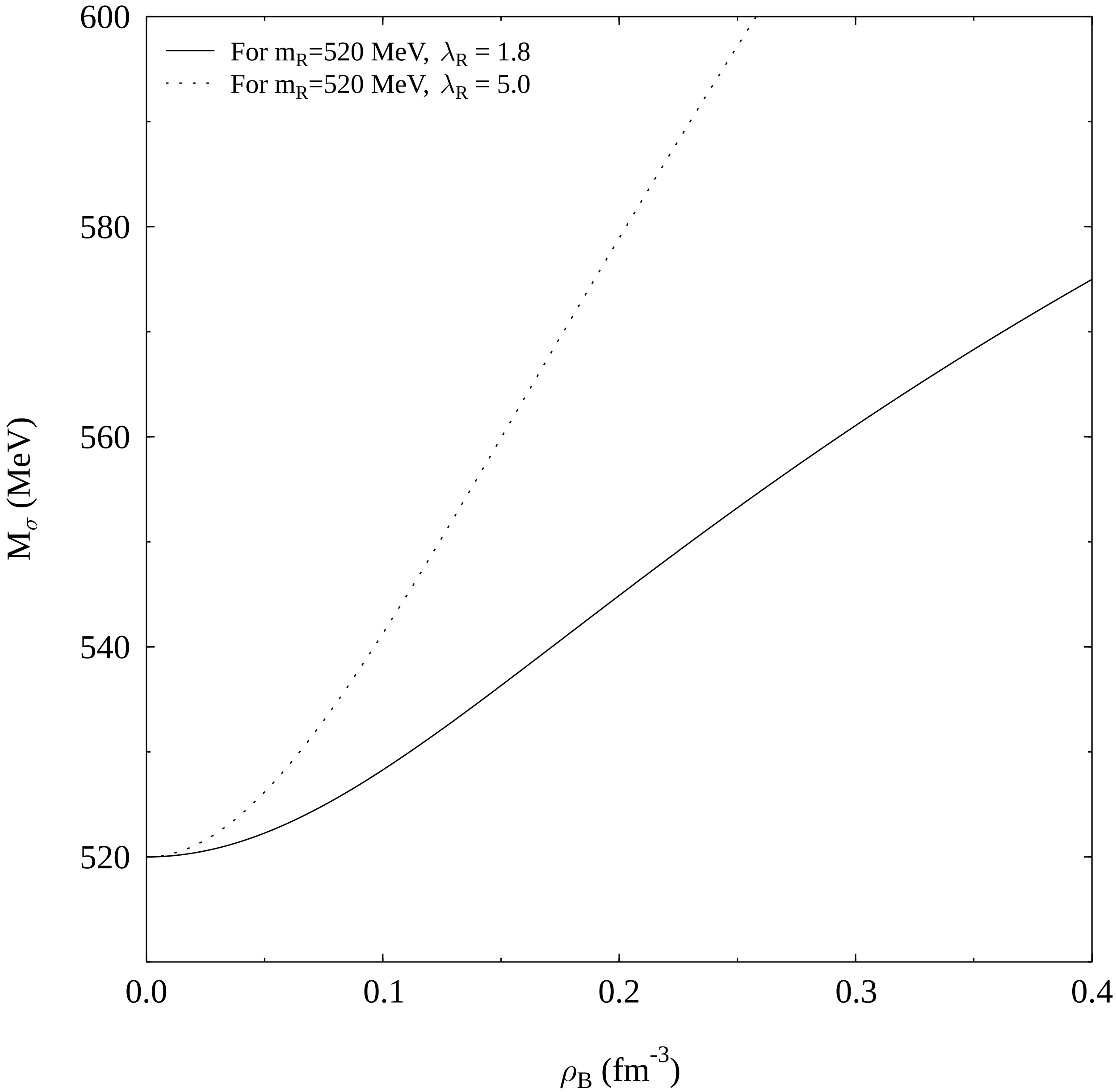

For m_R=520 MeV, λ_R = 1.8
For m_R=520 MeV, λ_R = 5.0
600
580
560
540
520
M_σ (MeV)
0.0
0.1
0.2
0.3
0.4
ρ_B (fm^{-3})

Figure 5

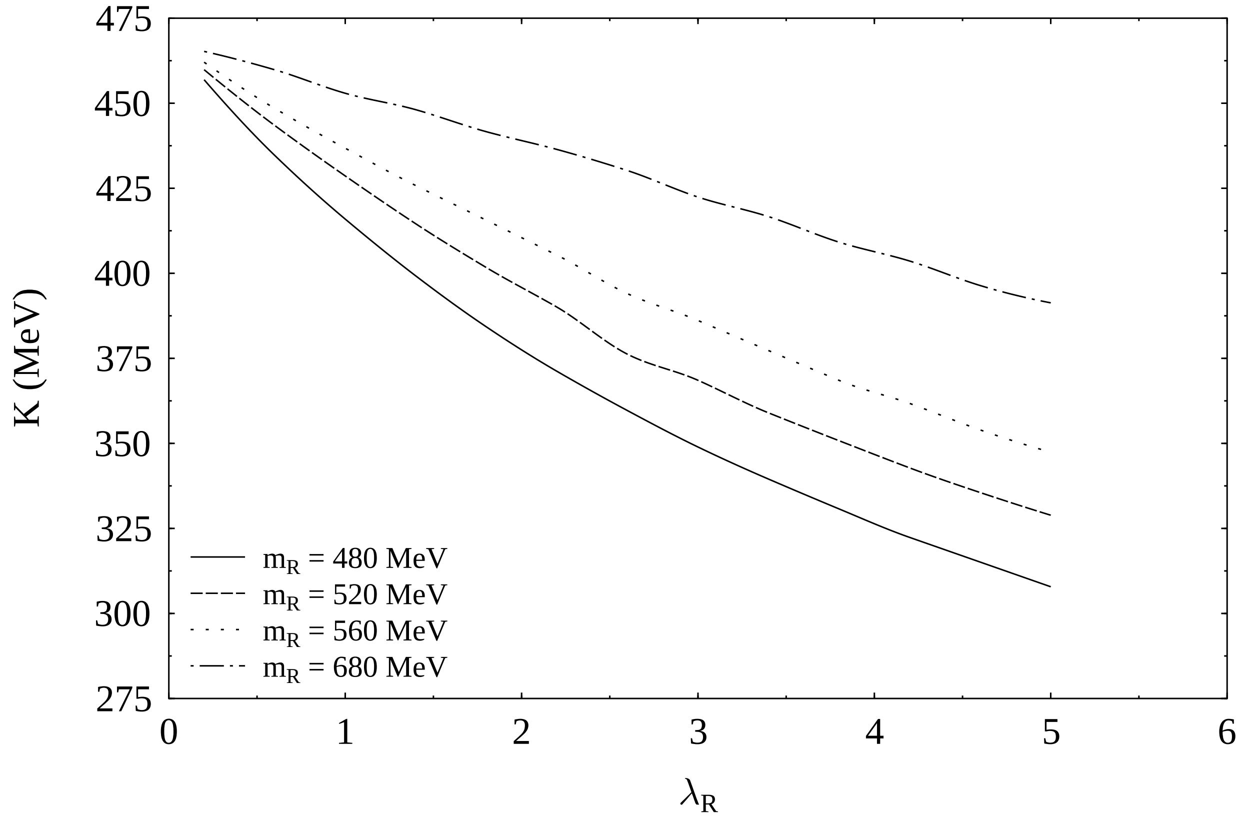


Figure 6